ZIAUDDIN UNIVERSITY
EXAMINATION BOARD

Mathematics Paper 3

Structured Questions

Model Paper 2025

Time Allowed: 2 hours
Total Marks: 75

You must answer on the question paper.

You must bring a soft pencil (preferably type B or HB), a clean eraser, and a dark blue or black pen. You will also need geometrical instruments.

Calculators are allowed.

Before attempting the paper, write your name, candidate number, centre name, and centre number clearly in the designated spaces.

Instructions for Candidates

- Answer all questions.
- Write your answer to each question in the space provided.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- You must show all necessary working clearly.
- Do not use an erasable pen or correction fluid.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Avoid writing over any barcodes printed on the paper.

Information for Candidates

- This paper consists of a total of **75 marks**.
- The number of marks assigned for every question or its parts is indicated within brackets [ ].
- A formula sheet will be provided with this paper.

Please read all questions carefully and follow the instructions exactly to ensure your responses are properly evaluated.

1

(a) Simplify:

$$\frac{2x^2 - 8}{x^2 - 4}$$

.... [2]

(b) Solve.

$$2x^2 - 5x - 3 = 0$$

.... [3]

(c)

The graph of $y = | x - 2 |$ and the line $y = x - 4$ are drawn on the same axes.
Explain, using algebra or otherwise, whether the two graphs intersect.

....

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.... **[3]**

(d) Describe the transformation that maps $y = f(x)$ onto $y = f(x - 3) - 2$.

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....

....

.... **[2]**

2

(a) The nth term of a sequence is $u_n = 3n - 2$. Find u_{20} and S_{20}.

$u_{20} =$ ....

$S_{20} =$ .... **[3]**

(b) A geometric sequence has first term 128 and common ratio $\frac{3}{4}$.

Find the least value of n for which $u_n < 10$.

$n =$ .... **[3]**

(c) Explain why the series $1 + \frac{1}{3} + \frac{1}{9} + \cdots$ has a finite sum, and find it.

....

....

....

....

....

[3]

3

(a) Prove the identity:

$$\sin(2x) = 2\ \sin x \cos x$$

[3]

(b) Solve:

$$\sin x = \cos x \text{ for } 0° \leq x < 360°.$$

x_1=...

x_2=... **[2]**

(c) A building 50 m away is viewed at an elevation of 32°.
Find its height.

height = ... **[3]**

(d) Explain how measurement error in the angle would affect your answer in (c).

....

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.... **[3]**

4

(a) Find the equation of the line L_1 passing through $A(2,3)$and $B(8,6)$ in the slope-intercept form.

....**[2]**

(b) Find the centre and radius of the circle $x^2 + y^2 - 4x - 6y + 9 = 0$.

$Centre$ = **(..................... , ...)**

$Radius$ = **.. [2]**

(c) In exact form, Find the coordinates where L_1 meets the circle.

.... **[3]**

(d) Determine whether $A(2,3)$ lies inside, on, or outside the circle.

....

....

.... **[3]**

5

(a) Differentiate $y = 3x^3 - 5x^2 + 2x - 4$.

$y' =$ .... **[2]**

(b) Find the x-coordinates of the stationary points of the function in part (a) and determine their nature.

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....

....

.... **[4]**

(c) Evaluate $\int_0^3 (x^2 - 4x + 5)\ dx$.

.... **[3]**

(d) A particle moves along a line such that its velocity $v = 6t - 2t^2$.
Find the time when the velocity is maximum, and state that maximum value.

$v =$ .... **[4]**

6

(a) Solve $3^x = 20$.

$x =$ .... **[2]**

(b) For $P = 500e^{0.06t}$, find t when $P = 2000$.

$t =$ .... **[3]**

(c) Explain two limitations of this model for very large t.

1. ....

....

2. ....

.... **[2]**

(d) From $e^{-kT} = \frac{1}{2}$ show $k = \frac{\ln 2}{T}$ and find k when $T = 30$.

k = **.. [4]**

7

(a) Points $A(1,2,3)$and $B(4,6,9)$ are given.

Find the vector $\overrightarrow{AB}$ and its magnitude.

$\overrightarrow{AB}$ = ...

$|\overrightarrow{AB}|$ = ...

[2]

(b) Find a unit vector in the direction of $\overrightarrow{AB}$.

$\hat{u}$ = ...**[2]**

(c) Find the acute angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$, where $C(2,5,7)$.

$\theta =$ ....**[3]**

(d) Express $z = 4(\cos 60^\circ + i\sin 60^\circ)$ in Cartesian form.

$z =$ ....**[2]**

(e) Explain geometrically what effect multiplying a complex number by i has on its position in the Argand diagram

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[2]

Model Paper 3 Marking Scheme

<table>
<tr><td colspan="2">Question</td><td>Solution</td><td>Notes</td></tr>
<tr><td rowspan="4">Q1</td><td>(a) Simplify:
$$\frac{2x^2-8}{x^2-4}$$</td><td>$$\frac{2x^2-8}{x^2-4}=\frac{2(x^2-4)}{x^2-4}=2$$</td><td>B1 for correct factorization of numerator and denominator
B1 for correct simplification</td></tr>
<tr><td>(b) Solve $2x^2-5x-3=0$</td><td>Discriminant: $\Delta=(-5)^2-4(2)(-3)=25+24=49$.
$$x=\frac{5\pm\sqrt{49}}{4}=\frac{5\pm7}{4}.$$
$$x=3 \text{ or } x=-\frac{1}{2}$$
(students can also use factorization)</td><td>M2 for using either factorization or quadratic formula (or any other algebraic method) correctly
A1 for both correct roots</td></tr>
<tr><td>(c) The graph of $y=\mid x-2\mid$ and the line $y=x-4$ are drawn on the same axes.
Explain, using algebra or otherwise, whether the two graphs intersect.</td><td>$$|x-2|=x-4$$
For equality, $x-4\geq0\Rightarrow x\geq4$.
For $x\geq4$,
$$|x-2|=x-2$$
Then
$$x-2=x-4\Rightarrow-2=0,$$
contradiction.
Hence, no real solution.
Therefore, the two graphs do not intersect.
(Graphically, $y=\mid x-2\mid$ lies entirely above $y=x-4$.)</td><td>B1 for considering correct case or inequality
B1 for showing algebraic contradiction or logical reasoning leading to “no solution.”
B1 for concluding correctly: graphs do not intersect (supported by algebraic or graphical argument).</td></tr>
<tr><td>(d) Describe the transformation that maps $y=f(x)$ onto $y=f(x-3)-2$.</td><td>$y=f(x-3)-2$ represents a translation of
• 3 units to the right, and
• 2 units downward.</td><td>B1 for correct interpretation of horizontal translation
B1 for correct interpretation of vertical translation</td></tr>
<tr><td>Q2</td><td>The nth term of a sequence is $u_n=3n-2$.
Find u_{20} and S_{20}.</td><td>$$u_{20}=3(20)-2=60-2=58$$
$$u_1=3(1)-2=1$$
$d=3$.
First term $u_1=3(1)-2=1$.
Common difference $d=3$.
$S_n=\frac{n}{2}(2a+(n-1)d)$.
$$S_{20}=\frac{2}{20}(2\cdot1+19\cdot3)$$
$=10(2+57)=10(59)=590$.</td><td>M1 for using correct formulae and doing correct substitution
A1 for correct value of the first term
A1 for the correct value of the sum</td></tr>
</table>

	(b) A geometric sequence has first term 128 and common ratio $\frac{3}{4}$. Find the least value of n for which $u_n < 10$.	General term: $u_n = 128(\frac{3}{4})^{n-1}$. Required: $128(\frac{3}{4})^{n-1} < 10$. Divide both sides: $(\frac{3}{4})^{n-1} < \frac{10}{128} = \frac{5}{64} = 0.078125$. Take natural logs (or trial): $(n-1)\ln(\frac{3}{4}) < \ln(0.078125)$. Since $\ln(3/4) < 0$, dividing reverses inequality: $n - 1 > \frac{\ln(0.078125)}{\ln(3/4)}$. Compute approx: $\ln(0.078125) \approx -2.5510$, $\ln(3/4) \approx -0.28768$. So RHS ≈ $(-2.5510)/(-0.28768) \approx 8.871$. Thus $n - 1 > 8.871$ $\Rightarrow n > 9.871$. Least integer $n = 10$. (Trial: $(3/4)^8 \approx 0.1001 > 0.0781$, $(3/4)^9 \approx 0.0751 < 0.0781$ $\Rightarrow n - 1 = 9$ gives $n = 10$.) Answer: $n = 10$	**M1** for setting up the correct inequality **M1** for using a valid method to solve, e.g. logs or numerical trial, handling sign reversal correctly **A1** for correct least integer
	(c) Explain why the series $1 + \frac{1}{3} + \frac{1}{9} + \cdots$ has a finite sum, and find it.	$1 + \frac{1}{3} + \frac{1}{9} + \cdots$ is geometric with first term $a = 1$ and common ratio $r = \frac{1}{3}$. Since $\|r\| = \frac{1}{3} < 1$, the terms become progressively smaller and approach zero, so the infinite series converges to a finite limit. $S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$.	**B1** for recognizing the series as geometric **B1** for explaining convergence **A1** for application of correct formula
Q3	(a) Prove the identity: $\sin(2x) = 2\ \sin x \cos x$	From the addition formula: $\sin(A+B) = \sin A \cos B + \cos A \sin B$. Let $A = B = x$, then $\text{Sin}(2x) = \sin(x + x)$ $= \sin x\cos x + \cos x\sin x$ $= 2\sin x\cos x$.	**B1** for using the addition formula correctly **M1** for substituting $A = B = x$ and simplifying correctly. **A1** for arriving at final identity

	(b) Solve $\sin x = \cos x$ for $0° \le x < 360°$.	Divide both sides by $\cos x$(where defined): $\tan x = 1$. $x = 45°, 225°$.	**M1** for using the correct trigonometric relationship **A1** for both correct angles in the interval
	(c) A building 50 m away is viewed at an elevation of 32°. Find its height.	$\tan 32° = \frac{h}{50}$ $h = 50\tan(32°)$ $h = 50(0.6249)$ = 31.2 m (to 3 s.f.)	**M1** for correct trigonometric ratio identified **M1** for correct substitution and rearrangement **A1** for correct numerical result
	(d) Explain how measurement error in the angle would affect your answer in (c).	Height depends on $\tan(\theta)$. For small angles, $\tan(\theta)$increases rapidly with θ. A small error in θ produces a large proportional error in height. Hence, if θ is overestimated, the calculated height will be too large; if underestimated, height will be too small.	**B1** recognizing that the height depends on $\tan(\theta)$. **B1** for using the fact that $\tan(\theta)$ changes rapidly for small/moderate θ **A1** for correctly identifying direction of error (overestimate ⇒ height too large, underestimate ⇒ height too small).
Q4	(a) Find the equation of the line L_1 passing through $A(2,3)$and $B(8,6)$ in the slope-intercept form.	Gradient $m = \frac{6-3}{8-2} = \frac{3}{6} = \frac{1}{2}$. Equation: $y - 3 = \frac{1}{2}(x - 2)$. Simplify: $y = \frac{1}{2}x + 2$.	**M1** for calculating correct gradient using the coordinates **A1** for final equation in any valid form
	(b) Find the centre and radius of the circle $x^2 + y^2 - 4x - 6y + 9 = 0$.	Completing the square $x^2 - 4x + y^2 - 6y + 9 = 0$ $(x-2)^2 - 4 + (y-3)^2 - 9 + 9 = 0$ $(x-2)^2 + (y-3)^2 = 4$. Centre: $(2,3)$ Radius: 2 (OR the student can use the general form of the equation of the circle: $x^2 + y^2 + 2gx + 2fy + c = 0$ And using centre = $(-g, -f)$ and radius = $\sqrt{g^2 + f^2 - c}$)	**M1** for either completing the squares of the *x* and *y* term OR rearranging the equation into the general form of the equation of the circle **A1** for correct centre and radius
	(c)In exact form, Find the coordinates where L_1meets the circle.	Line: $y = \frac{1}{2}x + 2$.	**M1** for substituting linear equation into

		Substitute into circle: $$x^2+(\frac{1}{2}x+2)^2-4x-6(\frac{1}{2}x+2)+9=0.$$ Simplify: $x^2+\frac{1}{4}x^2+2x+4-4x-3x-12+9=0.$ Combine: $\frac{5}{4}x^2-5x+1=0.$ Multiply by 4: $5x^2-20x+4=0.$ Solve the quadratic equation: $$x_1=2+\frac{4}{\sqrt{5}},\ x_2=2-\frac{4}{\sqrt{5}}.$$ Then $y=\frac{1}{2}x+2$ $$\Rightarrow y_1=3+\frac{2}{\sqrt{5}},\ y_2=3-\frac{2}{\sqrt{5}}.$$ Intersections: $(2\pm\frac{4}{\sqrt{5}},\ 3\pm\frac{2}{\sqrt{5}})$.	circle correctly to form a quadratic in x. **M1** for solving quadratic correctly (method for discriminant and roots). **A1** for both correct intersection coordinates in surd form
	(d) Determine whether $A(2,3)$lies inside, on, or outside the circle.	Circle centre $(2,3)$, radius $r=2$. Distance from $A(2,3)$ to centre is 0. $0<2$, so point A is inside the circle.	**B1** for identifying method: compare distance from centre with radius. **M1** for calculating or stating distance **A1** for correct conclusion: point lies **i**nside the circle (not on or outside).
Q5	(a) Differentiate $y=3x^3-5x^2+2x-4$.	$y'=9x^2-10x+2.$	**M1** for correct application of power rule to each term **A1** for Correct simplified derivative
	(b)Find the x-coordinates of the stationary points of the function in part (a) and determine their nature.	. $y'=9x^2-10x+2.$ $$x=\frac{10\pm\sqrt{28}}{18}=\frac{5\pm\sqrt{7}}{9}.$$ Second derivative: $y''=18x-10.$ Evaluate: $x_1=\frac{5-\sqrt{7}}{9}\Rightarrow y''(x_1)=18x_1-10<0\Rightarrow$ local **maximum**. $$x_2=\frac{5+\sqrt{7}}{9}$$ $\Rightarrow y''(x_2)=18x_2-10>0\Rightarrow$ local **minimum**.	**M1** for setting derivative = 0 **M1** for solving quadratic correctly (exact roots or correct decimals). **M1** for using second derivative sign chart to test nature of points **A1** for correct classification of both points

	(c) Evaluate $\int_0^3 (x^2 - 4x + 5)\ dx$.	$\int_0^2 (x2 - 4x + 5)\,dx$ $= \frac{x^3}{3} - 2x^2 + 5x + C.$ Evaluate from 0 to 3: $\left(\frac{27}{3} - 18 + 15\right) - 0$ $= (9 - 18 + 15) = 6.$	**M1** for integrating correctly **M1** for correct evaluation of limits **A1** for correct final answer
	(d) A particle moves along a line such that its velocity $v = 6t - 2t^2$. Find the time when the velocity is maximum, and state that maximum value.	$v(t) = 6t - 2t^2.$ Acceleration $a(t)$: $v'(t) = 6 - 4t.$ Set $a(t) = 0$ $\Rightarrow 6 - 4t = 0 \Rightarrow t = 1.5.$ Maximum velocity: $v(1.5) = 6(1.5) - 2(1.5)^2$ $= 9 - 4.5 = 4.5.$ $t = 1.5$(units), maximum $v = 4.5$(units)	**M1** for differentiating $v(t)$ **M1** for solving for t correctly **M1** for finding $v(1.5)$ correctly **A1** for correct final answer
Q6	(a) Solve $3^x = 20$.	Take logs: xln 3 = ln 20 $\Rightarrow x = \frac{\ln 20}{\ln 3} \approx 2.726.$	**M1** for taking logarithms correctly (or uses change of base). **A1** for correct expression (either in the exact ln fraction form, for decimal form upto three significant figures)
	(b) For $P = 500e^{0.06t}$, find t when $P = 2000$.	Take ln: ln 4 = $0.06t$ $\Rightarrow t = \frac{\ln 4}{0.06} \approx \frac{1.38629}{0.06}$ $\approx 23.1048.$	**M1** for correct equation set-up **M1** taking log on both sides and rearranging correctly **A1** for correct numeric value
	(c) Explain two limitations of this model for very large t.	Two reasonable limitations (any two valid points): • Exponential model assumes constant growth rate 0.06 and unlimited resources — unrealistic long term (carrying capacity ignored). • Ignores environmental changes, resource limits, disasters, migration or policy changes;	**B1** for first valid limitation (e.g. ignores carrying capacity). **B1** for second valid limitation (e.g. ignores other factors or assumes constant rate). *Accept any two clear, sensible limitations.*

		predictions for very large $t$become implausible.	
	(d) From $e^{-kT} = \frac{1}{2}$ show $k = \frac{\ln 2}{T}$ and find k when $T = 30$.	From $e^{-kT} = \frac{1}{2}$ take ln: $-kT = \ln(1/2) = -\ln 2$. $\Rightarrow k = \frac{\ln 2}{T}$. For $T = 30$: $k = \frac{\ln 2}{30} \approx \frac{0.693147}{30}$ ≈ 0.0231049.	**M1** for taking *ln* on both sides or equivalent **M1** for rearranging to make *k* the subject **M1** for correct substitution of value of T **A1** for correct numerical value to suitable accuracy *Accept any suitable order of the method steps*
Q7	(a) Points $A(1,2,3)$and $B(4,6,9)$are given. Find the vector $\overrightarrow{AB}$ and its magnitude.	$\overrightarrow{AB} = B - A$ $= (4-1, 6-2, 9-3) = (3,4,6)$. $\mid \overrightarrow{AB} \mid = 32 + 42 + 62$ $= 9 + 16 + 36 = \sqrt{61}$	**B1** for correct vector **B1** for correct magnitude *Accept decimal equivalent 7.81.* *Award B mark for vector even if magnitude omitted.*
	(b)Find a unit vector in the direction of $\overrightarrow{AB}$.	$\hat{u} = \frac{\overrightarrow{AB}}{\mid AB \mid} = \frac{1}{\sqrt{61}}(3,4,6)$ $\hat{u} = (\frac{3}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{6}{\sqrt{61}})$ $\approx (0.383, 0.512, 0.767)$	**M1** for dividing $\overrightarrow{AB}$ by its magnitude to form a unit vector **A1** for correct unit vector
	(c)Find the acute angle between $\overrightarrow{AB}$and $\overrightarrow{AC}$,where $C(2,5,7)$.	$\overrightarrow{AB} \cdot \overrightarrow{AC} = (3)(1) + (4)(3) + (6)(4)$ $= 3 + 12 + 24 = 39$ $\mid \overrightarrow{AC} \mid = \sqrt{1^2 + 3^2 + 4^2} = \sqrt{26}$, $\mid \overrightarrow{AB} \mid = \sqrt{61}$ $cos\theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\mid \overrightarrow{AB} \mid\mid \overrightarrow{AC} \mid}$ $= \frac{39}{\sqrt{61}\sqrt{26}} = \frac{39}{\sqrt{1586}}$ $\theta = \cos^{-1}(\frac{39}{\sqrt{1586}}) \approx 11.5°$	**M1** for finding $\overrightarrow{AC}$ correctly **M1** for substituting into $\cos\theta$ **A1** for correct final angle
	(d) Express $z = 4(\cos 60° + i\sin 60°)$in Cartesian form.	$z = 4(cos60° + isin60°)$ $4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ $z = 2 + 2\sqrt{3}i$	**M1** for correct values of sine and cosine and correct multiplication **A1** for correct Cartesian form

	(e) Explain geometrically what effect multiplying a complex number by i has on its position in the Argand diagram	Multiplying by $i$rotates any complex number **anticlockwise by** $90°$(or $\frac{\pi}{2}$radians) about the origin. The **magnitude (modulus)** of the complex number remains **unchanged**. Hence, the effect is a **pure rotation**, not a scaling or reflection. Example: $1 \to i,$ $i \to -1,$ $-1 \to -i,$ $-i \to 1.$ Therefore, on the Argand diagram, the point representing $z$moves **a quarter turn anticlockwise** around the origin.	**B1** for stating that multiplying by i rotates a complex number by $90°$**anticlockwise** about the origin. **B1** for recognizing that the **modulus remains unchanged** (a pure rotation, not a scaling). *Accept any clear geometric explanation mentioning "quarter-turn anticlockwise about the origin" or "rotation by* $\pi/2$ *radians".*